

Tunneling of Dirac Particles from a Double Myers-Perry Black Hole in Five Dimensions

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Abstract Motivated by the fermions tunneling formalism of Kerner and Mann, we attempt to investigate quantum tunneling of Dirac particles from a five dimensions double Myers-Perry black hole, which describes the superposition of two Myers-Perry black holes, each with a single angular momentum parameter and both in the same plane. After introducing several appropriate Gamma matrixes for the covariant Dirac equation, we obtain the tunneling probability of Dirac particles from the double Myers-Perry black hole, which gives the expected emission temperature as the case of scalar particles that obtained by others.

Keywords Hawking radiation · Tunneling · Myers-Perry black hole · Dirac particles

1 Introduction

Since the first derivation of Stephen Hawking [1], many papers have appeared to correctly derive Hawking temperature via different methods [2–8]. Particularly recent several years, investigation on Hawking radiation once again begin a hot topic [9–36]. The reason partly arises from the fact that a deeper understanding of Hawking radiation may shed some lights on seeking the underlying quantum gravity and partly owing to the 2008 startup of CERN Large Hadronic Collider (LHC) [37], which may explore mini black holes by detecting Hawking radiation and as an exciting possibility of finding new physics in the high energy collision scenarios. Among these methods, one intuitively simple but physically profound derivation is the tunneling paradigm [9] that visualizes the source of radiation as tunneling from inside to outside of horizon. Until now there mainly are two approaches to discuss tunneling. One, first used by Parikh and Wilczek [9], is called as the Null Geodesic method and the other, first proposed by Agheben et al. [15, 16] is called as the Hamilton-Jacobi ansatz. Both methods share a common that the tunneling probability is determined by the semi-classical WKB approximation, which relates the imaginary part of classical action

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that is related to the Boltzmann factor for emission at the Hawking temperature with the tunneling rate as $\Gamma \sim \exp(-2 \text{Im } W/\hbar)$, where W is the classical action of the trajectory to leading order in \hbar . However for the Null Geodesic method, one should solve the Geodesic equation and integral the radial momentum P_r for the radiant particles to get its classical action. And for the Hamilton-Jacobi ansatz, one needs to solve the relativistic Hamilton-Jacobi equation via performing variable separation for the action. The tunneling method now is shown very robust and has been successfully extended to a series black holes and even black rings. And people have believed that Hawking tunneling radiation provides not only an alternative conceptual means for understanding the actual emission process of black hole but also a useful verification of black hole thermodynamics.

Recently another tunneling approach [38] that involves dealing with the tunneling of fermions appeared. This method is thought can enforce the consistency of tunneling theory. Because a black hole can radiate all types of particles and the true emission spectrum contain contributions of both scalar particles and fermions with all spins. As the case of the scalar particles, the tunneling rate is also determined by the WKB approximation. Nevertheless, here one should employ the Dirac equation to find the action, and for different background space time, one should choose different γ^μ matrices to solve the Dirac equation. Now this method has been extended to the three dimension BTZ [39] and four dimension stationary space time [40] by picking up the γ^μ matrices appropriately. However there isn't literature except [41] to discuss the cases of five dimension and even high dimension space time. Because space time with different horizon topology and different dimensions, the γ^μ matrices will take different form.

In this paper we attempt to extend the fermions tunneling formalism to a five dimensions double Myers-Perry black hole, which might be of interest in studying spin-spin interactions in five dimensional general relativity. This black hole describes the superposition of two Myers-Perry black holes, each with a single angular momentum parameter and both in the same plane [42]. The black holes live in a background geometry which is the Euclidean C-metric with an extra flat time direction. This background possesses conical singularities in two adjacent compact regions, each corresponding to a set of fixed points of one of the $U(1)$ actions in the Cartan sub-algebra of $SO(4)$. On the other hand, great attentions recently have been paid to the Myers-Perry black hole. Because black holes from the LHC will decay in several phases [37], termed balding, spindown, Schwarzschild, and Planck, and it is assumed that the Myers-Perry black hole describes successfully a small, higher dimensional black hole during its spin-down phase. Investigations, particularly Hawking radiation, on this space time thus are of great importance and necessity. Until now however, there is no references to report Hawking tunneling radiation of Dirac particles across the Myers-Perry black hole as far as we know, hence we are interested in a five dimensions double Myers-Perry black hole. After choosing the γ^μ matrices appropriately in this five dimensions space time, we obtain the expected emission temperature.

The remainders of this paper are organized as follows. In Sect. 2, we introduce the five dimensions double Myers-Perry black hole and its basic properties. Then in Sect. 3, Hawking radiation of Dirac particles via tunneling from the double Myers-Perry black hole is discussed, and to choose the γ^μ matrices conveniently, the metric is rewritten. Section 4 is devoted to our discussion and conclusion.

2 Five Dimensions Double Myers-Perry Black Hole

In D space-time dimensions, the Belinskii-Zakharov method (or inverse scattering method) [43] can be used to construct new Ricci flat metrics with $D - 2$ commuting Killing vector

fields from known ones, by using purely algebraic manipulations. Such metrics can always be written in the form

$$ds^2 = G_{ab}(\rho, z)dx^a dx^b + e^{2v(\rho,z)}(d\rho^2 + dz^2), \tag{1}$$

where $a, b = 1, 2, 3, \dots, D$. In what follows we shall specialize all results to the case $D = 5$. By adopting a seed metric and performing 4-soliton transformation, after a series of algebra manipulations, the metric of the five dimensions double Myers-Perry black hole can be expressed as [42]

$$ds^2 = -\frac{H_y}{H_x} \left[dt + \left(\frac{\omega_\phi}{H_y} - q \right) d\phi \right]^2 + \frac{H_x}{H_y} \frac{\rho^2 u_3}{u_2 u_5} d\phi^2 + \frac{u_2 u_5}{u_3} d\psi^2 + k \frac{H_x}{F} (d\rho^2 + dz^2), \tag{2}$$

where k is an integration constant, $u_k = \pm \sqrt{\rho^2 + (z - a_k)^2} - (z - a_k)$ refer to soliton positions in the BZ method: in which the “+” pole refers to a soliton and is denoted by u_k ; the “-” pole refers to an anti-soliton and is denoted by $-u_k$. The metric functions are

$$H_x = M_0 + b^2 M_1 + c^2 M_2 + bc M_3 + b^2 c^2 M_4, \tag{3}$$

$$H_y = \frac{\rho^2}{u_2 u_5} \left[M_0 \frac{u_1 u_4}{\rho^2} - b^2 M_1 \frac{u_4}{u_1} - c^2 M_2 \frac{u_1}{u_4} - bc M_3 + b^2 c^2 M_4 \frac{\rho^2}{u_1 u_4} \right], \tag{4}$$

$$\omega_\phi = 2 \sqrt{\frac{u_3}{u_2 u_5}} [b R_1 \sqrt{M_0 M_1} + c R_4 \sqrt{M_0 M_2} - b^2 c R_4 \sqrt{M_1 M_4} - b^2 c R_1 \sqrt{M_2 M_4}], \tag{5}$$

where $R_i = \sqrt{\rho^2 + (z - a_i)^2}$ and the functions M_i are

$$M_0 = u_2 u_3^2 u_5 (u_1 - u_4)^2 (\rho^2 + u_1 u_2)^2 (\rho^2 + u_1 u_5)^2 (\rho^2 + u_4 u_2)^2 (\rho^2 + u_4 u_5)^2, \tag{6}$$

$$M_1 = u_1^2 u_2^2 u_3 u_5^2 (u_1 - u_3)^2 (\rho^2 + u_1 u_4)^2 (\rho^2 + u_2 u_4)^2 (\rho^2 + u_4 u_5)^2, \tag{7}$$

$$M_2 = u_2^2 u_3 u_4^2 u_5^2 (u_3 - u_4)^2 (\rho^2 + u_1 u_2)^2 (\rho^2 + u_1 u_4)^2 (\rho^2 + u_1 u_5)^2, \tag{8}$$

$$M_3 = 2u_1 u_2^2 u_3 u_4 u_5^2 (u_1 - u_3)(u_3 - u_4) (\rho^2 + u_1)^2 (\rho^2 + u_4)^2 (\rho^2 + u_1 u_2) \times (\rho^2 + u_1 u_5) (\rho^2 + u_2 u_4) (\rho^2 + u_4 u_5), \tag{9}$$

$$M_4 = u_1^2 u_2^2 u_4^2 u_5^2 (u_1 - u_3)^2 (u_1 - u_4)^2 (u_3 - u_4)^2. \tag{10}$$

Moreover

$$F = u_3^3 (u_1 - u_4)^2 (\rho^2 + u_1 u_2) (\rho^2 + u_1 u_4)^2 (\rho^2 + u_1 u_5) (\rho^2 + u_2 u_5)^2 (\rho^2 + u_2 u_4) \times (\rho^2 + u_4 u_5) \prod_{i=1}^5 (\rho^2 + u_i^2) / [(\rho^2 + u_1 u_3) (\rho^2 + u_2 u_3) (\rho^2 + u_3 u_4) (\rho^2 + u_4 u_5)]. \tag{11}$$

The metric (2) is invariant under the exchange

$$(a_1, a_2, b) \leftrightarrow (a_4, a_5, c), \tag{12}$$

where b, c are two parameters.

It is clear that the metric (2) gives a six parameter family of solution, which physically can be taken to be the two black hole masses and angular momentum, together with the two

conical singularities. The eigenvector of the two timelike rods gains a spatial component, along the φ -direction. These new components are the angular velocities of the individual black hole horizons, which respectively take the form as

$$\Omega_1^\phi = \frac{a_{41}b}{a_{51}\Delta}, \quad \Omega_2^\phi = \frac{a_{54}\bar{b} + a_{51}\bar{c}}{a_{41}a_{51}\bar{\Delta}}, \tag{13}$$

where, for convenience, we have introduced the quantities

$$\Delta = 2a_{21} \frac{a_{31}}{a_{51}} b^2, \quad \bar{\Delta} = 2a_{54} + \frac{(\bar{b} + \bar{c})(a_{54}\bar{b} + a_{51}\bar{c})}{a_{41}^2 a_{51}}, \quad \bar{b} = a_{31}b, \bar{c} = a_{43}c. \tag{14}$$

These angular velocities reduce to the horizon angular velocities of single Myers-Perry black holes in the limits $a_3 = a_4 = a_5$ and $a_1 = a_2 = a_3$, respectively. By taking these limits, one can see that the solution (2) contains two Myers-Perry black holes [42]. The ADM mass, ADM angular momentum, horizon angular velocity, area and temperature of the two Myers-Perry black holes are given, respectively, by [42]

$$\begin{aligned} M_1^{Komar} &= \frac{3\pi}{8} \Delta_1, & j_1^\phi &= \frac{\pi b}{4} \Delta_1, & \Omega_1^\phi &= \frac{b}{\Delta_1}, \\ A_1 &= 2\pi^2 \sqrt{2a_{21}} \Delta_1, & T_1 &= \frac{1}{2\pi} \frac{\sqrt{2a_{21}}}{\Delta_1}, \end{aligned} \tag{15}$$

and

$$\begin{aligned} M_2^{Komar} &= \frac{3\pi}{8} \Delta_2, & j_1^\phi &= \frac{\pi c}{4} \Delta_2, & \Omega_2^\phi &= \frac{c}{\Delta_2}, \\ A_2 &= 2\pi^2 \sqrt{2a_{54}} \Delta_2, & T_1 &= \frac{1}{2\pi} \frac{\sqrt{2a_{54}}}{\Delta_2}, \end{aligned} \tag{16}$$

where

$$\Delta_1 = 2a_{21} + b^2, \quad \Delta_2 = 2a_{54} + c^2. \tag{17}$$

This is the reason that why we call metric (2) the double Myers-Perry black hole.

3 Tunneling of Dirac Particles

In this section, we focus on studying Hawking radiation of Dirac particles via tunneling from the five dimensions double Myers-Perry black hole. For the sake of discussing conveniently, we take

$$\begin{aligned} P &= -\frac{\rho^2 u_3}{u_2 u_5} \Big/ \frac{H_x}{H_y} \frac{\rho^2 u_3}{u_2 u_5} - \frac{H_y}{H_x} \left(\frac{\omega_\phi}{H_y} - q \right)^2, & Q &= \frac{F}{k H_x}, & E &= \frac{u_2 u_5}{u_3}, \\ M &= \frac{H_x}{H_y} \frac{\rho^2 u_3}{u_2 u_5} - \frac{H_y}{H_x} \left(\frac{\omega_\phi}{H_y} - q \right)^2, \\ N &= -\frac{H_y}{H_x} \left(\frac{\omega_\phi}{H_y} - q \right) \Big/ \frac{H_x}{H_y} \frac{\rho^2 u_3}{u_2 u_5} - \frac{H_y}{H_x} \left(\frac{\omega_\phi}{H_y} - q \right)^2. \end{aligned} \tag{18}$$

The new form of metric (2) now takes the form as

$$ds^2 = -P dt^2 + \frac{1}{Q} d\rho^2 + M(d\phi + N dt)^2 + E d\psi^2 + \frac{1}{Q} dz^2. \tag{19}$$

At the event horizon ρ_h , the coefficients in (18) obey

$$P(\rho_h) = Q(\rho_h) = 0, \quad N(\rho_h) = \Omega_h, \tag{20}$$

where Ω_h is the angular velocity at the event horizon.

To get the tunneling rate as mentioned above, the major problem is how to find the action of emission. Until now, one can adopt the null geodesic equation and relativistic Hamilton-Jacobi equation to finish it. As far as the Dirac particles are concerned in this paper, we restore the following Dirac equation

$$i\gamma^\nu e_\nu^\mu D_\mu \psi - \frac{m}{\hbar} \psi = 0, \tag{21}$$

where D_μ being the spinor covariant derivative defined by $D_\mu = \partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{[a}\gamma_{b]}$ and ω_μ^{ab} is the spin connection corresponding to the tetrad e_ν^μ while m is the mass of Dirac particle. Based on the relation $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I$, the Gamma matrix in this background can be chosen as

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, & \gamma^1 &= \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, & \gamma^4 &= \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}, \\ \gamma^3 &= \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, & \gamma^2 &= \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \end{aligned} \tag{22}$$

in which σ^i ($i = 1, 2, 3$) is the general Pauli Sigma matrixes that take the form as

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{23}$$

According to metric (19), the tetrad fields e_ν^μ can be selected as

$$\begin{aligned} e_0^\mu &= \left(\frac{1}{\sqrt{P}}, 0, 0, 0, -\frac{N}{\sqrt{P}} \right), \\ e_1^\mu &= (0, \sqrt{Q}, 0, 0, 0), \\ e_2^\mu &= \left(0, 0, \frac{1}{\sqrt{M}}, 0, 0 \right), \\ e_3^\mu &= \left(0, 0, 0, \frac{1}{\sqrt{E}}, 0 \right), \\ e_4^\mu &= (0, 0, 0, 0, \sqrt{Q}). \end{aligned} \tag{24}$$

These are insufficient to solve the Dirac equation, we hence employ the following ansatz

$$\psi_\uparrow(t, \rho, \phi, \psi, z) = \begin{bmatrix} A(t, \rho, \phi, \psi, z)\xi_\uparrow \\ B(t, \rho, \phi, \psi, z)\xi_\uparrow \end{bmatrix} \exp\left[\frac{i}{\hbar} I_\uparrow(t, \rho, \phi, \psi, z)\right], \tag{25}$$

$$\psi_\downarrow(t, \rho, \phi, \psi, z) = \begin{bmatrix} C(t, \rho, \phi, \psi, z)\xi_\downarrow \\ D(t, \rho, \phi, \psi, z)\xi_\downarrow \end{bmatrix} \exp\left[\frac{i}{\hbar} I_\downarrow(t, \rho, \phi, \psi, z)\right], \tag{26}$$

where $\xi_{\uparrow/\downarrow}$ is the eigenvectors of σ^3 and $I_{\uparrow/\downarrow}$ is the action of the radiant spin particles. In this section, we are only interested in the spin up case since the spin down is fully similar to this other than some changes of the sign. Inserting (25) into the Dirac Equation, after dividing the exponential term and multiplying \hbar , we find to leading order in \hbar

$$B\left(\frac{\partial_t I_{\uparrow}}{\sqrt{P}} + \sqrt{Q}\partial_{\rho}I_{\uparrow} - \frac{N\partial_{\phi}I_{\uparrow}}{\sqrt{P}}\right) + A(m - \sqrt{Q}\partial_z I_{\uparrow}) = 0, \tag{27}$$

$$B\left(\frac{1}{\sqrt{E}}\partial_{\psi}I_{\uparrow} + \frac{i}{\sqrt{M}}\partial_{\phi}I_{\uparrow}\right) = 0, \tag{28}$$

$$A\left(\frac{\partial_t I_{\uparrow}}{\sqrt{P}} - \sqrt{Q}\partial_{\rho}I_{\uparrow} + \frac{N\partial_{\phi}I_{\uparrow}}{\sqrt{P}}\right) - B(m + \sqrt{Q}\partial_z I_{\uparrow}) = 0, \tag{29}$$

$$A\left(\frac{1}{\sqrt{E}}\partial_{\psi}I_{\uparrow} + \frac{i}{\sqrt{M}}\partial_{\phi}I_{\uparrow}\right) = 0. \tag{30}$$

Note that the derivatives of A and B and the components ω_{μ} are all of the order \hbar , so they are neglected to the lowest order in WKB approximation. Considering the symmetries of the background spacetime, we carry out the following separation variable

$$I_{\uparrow} = -\omega t + W(\rho, \psi) + J(\phi) + L(z) + K, \tag{31}$$

where ω , J and L are all real constants which respectively represent the emitted particle’s energy and angular momentum, and K is a complex constant. Then we get

$$B\left(\frac{-\omega + JN}{\sqrt{P}} + \sqrt{Q}W_{\rho}\right) + A(m - \sqrt{Q}L) = 0, \tag{32}$$

$$B\left(\frac{1}{\sqrt{E}}W_{\psi} + \frac{i}{\sqrt{M}}J\right) = 0, \tag{33}$$

$$A\left(\frac{-\omega + JN}{\sqrt{P}} - \sqrt{Q}W_{\rho}\right) - B(m + \sqrt{Q}L) = 0, \tag{34}$$

$$A\left(\frac{1}{\sqrt{E}}W_{\psi} + \frac{i}{\sqrt{M}}J\right) = 0. \tag{35}$$

From (33) and (35), we find $W_{\psi} = -i\sqrt{E}J/\sqrt{M}$ regardless the explicit value of A or B , implying the action W_{ψ} would contribute to the tunneling rate of outgoing and ingoing modes. However, as we will discuss, the total tunneling probability is the ratio of tunneling probability of outgoing modes to ingoing modes, this contribution would be counteracted and thus can be ignored. And from (32) and (34), one can easily see the two equations have a non-trivial solution for A and B if and only if the determinant of the coefficient matrix vanishes, so we have

$$W_{\rho} = \pm \int \frac{\sqrt{(\omega - JN)^2 + Q(QL^2 - m^2)}}{\sqrt{PQ}}. \tag{36}$$

At the near horizon, the above equation can be expressed as

$$W_{\pm\rho} = \pm \int \frac{\sqrt{(\omega - J\Omega_h)^2 + Q'(\rho - \rho_h)[Q'(\rho - \rho_h)L^2 - m^2]}}{\sqrt{P'Q'}(\rho - \rho_h)}, \tag{37}$$

where $P' = \partial_\rho P|_{\rho=\rho_h}$, $Q' = \partial_\rho Q|_{\rho=\rho_h}$. The imaginary part of W_ρ can be calculated using the above equation. Integrating it at the horizon directly leads to

$$\text{Im } W_\rho = \pm i\pi \frac{\omega - J\Omega_h}{\sqrt{P'Q'}}, \tag{38}$$

where $+/-$ sign corresponds to outgoing/incoming solutions. So when particles tunnel across the horizon, the outgoing and ingoing rates are respectively given by

$$P(\text{out}) = \exp\left[-\frac{2}{\hbar}\text{Im}I_\uparrow\right] = \exp\left[-\frac{2}{\hbar}(\text{Im}W_{+\rho} + \text{Im}W_\psi + \text{Im}K)\right], \tag{39}$$

$$P(\text{in}) = \exp\left[-\frac{2}{\hbar}\text{Im}I_\uparrow\right] = \exp\left[-\frac{2}{\hbar}(\text{Im}W_{-\rho} + \text{Im}W_\psi + \text{Im}K)\right]. \tag{40}$$

Note that any particles classically enter the horizon with no barrier, implying the tunneling rate should be unity for incoming particles crossing the horizon, here it means $\text{Im } W_{-\rho} = -\text{Im}W_\psi - \text{Im}K$. The tunneling probability of Dirac particles crossing from inside to outside horizon is naturally written as

$$\Gamma \sim \frac{P(\text{out})}{P(\text{in})} = \exp(-4\text{Im}W_{+\rho}) = \exp\left(-4\pi \frac{\omega - J\Omega_h}{\sqrt{P'Q'}}\right), \tag{41}$$

as we have set \hbar to unity, which will give the expected Hawking temperature. We of course also can get the same emission temperature of spin down particles as (21) inserts into (26).

4 Conclusion

In this paper we have discussed tunneling of Dirac particles from an asymptotically flat, vacuum solution of five dimensional general relativity describing two Myers-Perry black holes, each with a singular angular momentum parameter, and both in the same plane. We obtained the expected Hawking temperature by choosing five appropriate Gamma matrices. The correction to the emission temperature of course also can be given as the self-gravitational interaction and back reaction of radiant Dirac particles are taken into account. Because in this case, when particle tunnels out with energy ω , the black hole mass will reduce to $M - \omega$ and the action in (36) correspondingly will be modified.

The fermions tunneling method has been extended to a series black holes and black rings as mentioned in instruction, there however is not literature to pay attention to the high dimensional space time. The major problems we think also arises from the choosing of Gamma matrices and our next job is to find a series Gamma matrices to extend this method to high dimensional space time, particularity to that with many rotating axis, to perfect the fermions tunneling method.

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References

1. Hawking, S.W.: Math. Phys. **43**, 199 (1975)
2. Damour, T., Ruffini, R.: Phys. Rev. D **14**, 332 (1976)

3. Chandrasekhar, S.: Proc. R. Soc. Lond. A **349**, 541 (1976)
4. Sannan, S.: Gen. Relativ. Gravit. **20**, 239 (1988)
5. Gibbons, G.W., Perry, M.J.: Proc. R. Soc. Lond. A **358**, 467 (1978)
6. Hartle, J.B., Hawking, S.W.: Phys. Rev. D **13**, 2188 (1976)
7. Gibbons, G.W., Hawking, S.W.: Phys. Rev. D **15**, 2738 (1977)
8. Boulware, D.: Phys. Rev. D **11**, 1404 (1975)
9. Parikh, M.K., Wilczek, F.: Phys. Rev. Lett. **85**, 5042 (2000)
10. Zhang, J., Zhao, Z.: Phys. Lett. B **618**, 14 (2005)
11. Zhang, J.Y., Zhao, Z.: Nucl. Phys. B **725**, 173 (2005)
12. Yang, S.Z., Jiang, Q.Q., Li, H.L.: Chin. Phys. **14**, 2411 (2005)
13. Han, Y.W.: Chin. Phys. **16**, 923 (2007)
14. Zhang, J.Y., Zhao, Z.: Mod. Phys. Lett. A **21**, 1865 (2006)
15. Agheben, M., Nadalini, M., Vanzo, L., Zerbini, S.: J. High Energy Phys. **0505**, 014 (2005)
16. Medved, A.J.M., Vagenas, E.: Mod. Phys. Lett. A **20**, 2449 (2005)
17. Yang, S.Z.: Int. J. Theor. Phys. **45**, 965 (2006)
18. Jiang, Q.Q., Wu, S.Q.: Phys. Lett. B **635**, 151 (2006)
19. Jiang, Q.Q., Wu, S.Q., Cai, X.: Phys. Rev. D **76**, 064003 (2006)
20. Vagenas, E.C.: Phys. Lett. B **503**, 399 (2001)
21. Vagenas, E.C.: Phys. Lett. B **533**, 302 (2002)
22. Vagenas, E.C.: Phys. Lett. A **17**, 609 (2002)
23. Vagenas, E.C.: Phys. Lett. B **559**, 65 (2003)
24. Vagenas, E.C.: Phys. Lett. B **584**, 127 (2004)
25. Zeng, X.X., Hou, J.S., Yang, S.Z.: Pramana J. Phys. **70**, 409 (2008)
26. Liu, C.Z., Zhang, J.Y., Zhao, Z.: Phys. Lett. B **639**, 670 (2006)
27. Liu, W.B.: Phys. Lett. B **634**, 541 (2006)
28. Chen, D.Y., Yang, S.Z.: Gen. Relativ. Gravit. **39**, 1503 (2007)
29. Zeng, X.X., Lin, K., Yang, S.Z.: Int. J. Theor. Phys. **47**, 2533 (2008)
30. Jiang, Q.Q., Wu, S.Q.: Phys. Lett. B **647**, 200 (2007)
31. Jiang, Q.Q., Wu, S.Q., Cai, X.: Phys. Lett. B **651**, 58 (2007)
32. Jiang, Q.Q., Wu, S.Q., Cai, X.: Phys. Lett. B **651**, 65 (2007)
33. Robinson, S.P., Wilczek, F.: Phys. Rev. Lett. **95**, 011303 (2005)
34. Banerjee, R., Kulkarni, S.: Phys. Rev. D **77**, 024018 (2008)
35. Iso, S., Umetsu, H., Wilczek, F.: Phys. Rev. Lett. **96**, 151302 (2006)
36. Murata, K., Soda, J.: Phys. Rev. D **74**, 044018 (2006)
37. Giddings, S.B.: [arXiv:hep-th/0709.1107](https://arxiv.org/abs/hep-th/0709.1107) (2007)
38. Kerner, R., Mann, R.B.: Class. Quantum Gravity **25**, 095014 (2008)
39. Li, R., Ren, J.R.: Phys. Lett. B **661**, 370 (2008)
40. Chen, D.Y., Jiang, Q.Q., Zu, X.T.: Phys. Lett. B **665**, 106 (2008)
41. Jiang, Q.Q.: [arXiv:hep-th/0807.1358](https://arxiv.org/abs/hep-th/0807.1358) (2008)
42. Herdeiro, C.A.R., Rebelo, C., Zilhão, M., Costa, M.S.: [arXiv:hep-th/0805.1206](https://arxiv.org/abs/hep-th/0805.1206) (2008)
43. Belinskii, V.A., Zakharov, V.E.: Sov. Phys. JETP **50**, 1 (1979)